Roll No. ....

W - 3201

Third Semester Examination 2021

M.Sc. (Mathematics)

Integration Theory and Functional Analysis (I)

Paper - I

Time :- 3 Hrs. M.M. 80

SECTION - A (4x3=12)

Very short answer type questions.(maximum 3 lines)

Q.1 Define signed measure ?

Q.2 What is Product measure ?

Q.3 Define Equivalent norms ?

Q.4 Define bounded linear transformation ?

SECTION - B

Short answer type questions with maximum word limit 150. (4x5=20)

Q.5 Prove that a union of any countably collection of positive set is positive ?

OR

State and prove Hahn Decomposition theorem ?

Q.6 State and prove Fubini's theorem ?

OR

Prove that every absolutely continuos function F defined on (a,b) is of bounded variation ?

Q.7 Let X be a normed linear space. The closed unit Ball $B=\{x \in X : ||x|| < 1\}$ in X is compact if and only if X is finite dimensional ?

P.T.O.

OR

Let X be a linear space and $||.||_1$ and $||.||_2$ be two norms defined on X then these norms on X are equivalent iff their exist a positive constant m and M such that

$m\ ||x||_1 < ||x||_2 < M\ ||x||_1\ \forall\ x \in X$

Q.8 Let X,Y be normed linear space and $T : X \rightarrow Y$ a linear transformation. Then the following conditions are equivalent

i) T is bounded.

ii) T is continuous.

iii) T is continuous at one point.

OR

Define strong and weak convergence. In a finite dimensional normed linear space, strong and weak convergence coincide ?

SECTION - C

Long answer type questions with maximum word limit 500. (4x12=48)

Q.9 State and prove Riesz Representation theorem ?

OR

State and prove Radon Nikodym theorem ?

Q.10 Prove that every compact Baire set is Gs type ?

OR

A function f is of bounded variation if and only if it can be expressed as a difference of two monotonic functions both non-decreasing ?

Q.11 A non-empty subset of the normed linear space K^n is compact if and only if it closed and bounded ?

OR

Let μ be a closed linear sub space of a nls X then the quotient space X/M is a nls with the norm

$|| x + \mu || = f\{||x+m|| : m \in M\}$

Further, if X is a Banach space then so is X/M.

Q.12 State and prove Fixed point theorem ?

OR

Let $1 < P < \infty$ and $\frac{1}{p} + \frac{1}{q} = 1$

then lp* is isometrically isemorphic to lq.

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